
PREPARING FOR THE QUADRATIC FORMULA

□ *THE GENERAL QUADRATIC EQUATION*

Consider a general quadratic equation in standard form:

$$ax^2 + bx + c = 0$$

where x represents the variable (the unknown) and a , b , and c are numbers. Notice that the squared term is first, the linear term is second, the constant term is third, and there's a 0 on the right side of the equation. The examples below show some quadratic equations along with their corresponding values of a , b , and c .

$3x^2 - 7x + 10 = 0$	$\longrightarrow$	$a = 3$	$b = -7$	$c = 10$
$y^2 - 9 = 0$	$\longrightarrow$	$a = 1$	$b = 0$	$c = -9$
$-4z^2 + 19z = 0$	$\longrightarrow$	$a = -4$	$b = 19$	$c = 0$
$2w^2 = 8 - 5w$	$\longrightarrow$	$a = 2$	$b = 5$	$c = -8$

To find the values of a , b , and c for this last equation, we need to rewrite the quadratic equation in standard form:

$2w^2 + 5w - 8 = 0$, from which the values of a , b , and c can be determined.

Homework

1. For each quadratic equation, first make sure that it's in standard form ($ax^2 + bx + c = 0$); then determine the values of a , b , and c :

a. $3x^2 + 9x + 17 = 0$

b. $2n^2 - 8n + 14 = 0$

c. $6y^2 + 2y - 2 = 0$

d. $5t^2 - 13t - 1 = 0$

e. $12a^2 + 13 = 0$

f. $x^2 - 13x = 0$

g. $u^2 - u + 1 = 0$

h. $-2w^2 - 19 = 0$

i. $-w^2 + 14w = 0$

j. $-z^2 - 99 = 0$

k. $2x^2 = 4x + 3$

l. $-3y^2 - 2y = -5$

m. $c^2 + 4 = 7c$

n. $6m^2 = 1 + m$

o. $18k^2 = 0$

p. $-3x^2 = -3x$

□ **ORDER OF OPERATIONS**

Remember that exponents are done before multiplication, which itself comes before any addition or subtraction. For example,

$$\begin{aligned}
 & 12^2 - 4(2)(3) \\
 = & 144 - 4(2)(3) && \text{(exponent first)} \\
 = & 144 - 24 && \text{(multiplication second)} \\
 = & \mathbf{120} && \text{(subtraction last)}
 \end{aligned}$$

For a second example,

$$\begin{aligned}
 & (-9)^2 - 4(3)(-5) \\
 = & 81 - 4(3)(-5) && \text{(exponent first)} \\
 = & 81 - (-60) && \text{(multiplication second)} \\
 = & 81 + 60 && \text{(subtracting a negative is adding)} \\
 = & 141 && \text{(addition last)}
 \end{aligned}$$

□ **SQUARE ROOTS**

$$\sqrt{36} = 6 \qquad -\sqrt{144} = -12$$

$$\sqrt{1} = 1 \qquad \sqrt{0} = 0$$

$$\sqrt{225} = 15 \qquad \sqrt{50} \approx 7.0711$$

$\sqrt{-9}$ does not exist (in our algebra class)  is approximately equal to

□ **OPPOSITES**

Recall that the *opposite* of N is $-N$. So we note the following:

- 1) The opposite of a positive number is a negative number; for example, $-(+7) = -7$. Thus, if $b = 7$, then $-b = -7$.
- 2) The opposite of a negative number is a positive number; for example, $-(-12) = 12$. Thus, if $b = -12$, then $-b = 12$.
- 3) The opposite of a squared quantity (that isn't 0) is negative; for example, $-9^2 = -81$. On the other hand, don't forget that $(-9)^2 = 81$.
- 4) The opposite of 0 is 0.

□ *PLUS/MINUS*

The symbol “ $\pm$ ” is read “*plus or minus*” and is just a short way of indicating two numbers at once. For example, if you want to indicate the two numbers 7 and -7 , you may write just ± 7 . Another example might be $\pm\sqrt{81}$, which stands for the two numbers 9 and -9 . But $\pm\sqrt{0}$ would only be 0, since $+0$ and -0 are really just two ways to express 0.

The equation $x^2 = 25$ has two solutions, namely that x could be 5 or -5 , so we could write the solution as $x = \pm 5$.

Let’s work out a specific problem containing the “plus or minus” sign.

Simplify: $\frac{10 \pm \sqrt{25}}{5}$

Taking the square root yields $\frac{10 \pm 5}{5}$.

Using the plus sign, we get $\frac{10+5}{5} = \frac{15}{5} = 3$,

while using the minus set yields $\frac{10-5}{5} = \frac{5}{5} = 1$.

Thus, the expression $\frac{10 \pm \sqrt{25}}{5}$ is merely a strange way to represent the numbers 3 and 1.

Homework

2. Evaluate each expression:

a. $13^2 - 4(3)(2)$

b. $0^2 - 4(2)(-1)$

c. $(-3)^2 - 4(-1)(-2)$

d. $(-5)^2 - 4(1)(0)$

e. $0^2 - 4(17)(0)$

f. $(-4)^2 - (-4)(-5)$

3. Evaluate each expression:

a. $\sqrt{49}$

b. $-\sqrt{6+3}$

c. $\sqrt{16} + \sqrt{9}$

d. $\sqrt{-25}$

e. $\sqrt{(-32)(-2)}$

f. $\sqrt{(-5)^2 - 4(1)(6)}$

g. $\sqrt{6^2 - 4(1)(9)}$

h. $\sqrt{(-10)^2 - 4(8)(2)}$

i. $\sqrt{1^2 - 4(1)(1)}$

j. $\sqrt{(-7)^2 - 4(4)(-2)}$

4. Evaluate each expression (the leading minus sign represents the *opposite* of the quantity which follows it):

a. $-(13)$

b. $-(-5)$

c. $-(-(-7))$

d. $-(+7)$

e. $-(-1)$

f. $-(0)$

5. What number or numbers are represented by each of the following?

a. ± 121

b. $\pm\sqrt{121}$

c. $\pm\sqrt{0}$

d. $\pm\sqrt{-1}$

6. Simplify each expression:

a. $100 + \sqrt{49}$

b. $-5 - \sqrt{121}$

c. $\frac{3 + \sqrt{16}}{2}$

d. $\frac{-8 - \sqrt{4}}{10}$

e. $7 - \sqrt{49}$

f. $-10 + \sqrt{100}$

g. $\frac{4 \pm \sqrt{16}}{6}$

h. $\frac{-1 \pm \sqrt{81}}{9}$

7. Evaluate the expression $b^2 - 4ac$ for the given values of a , b , and c :

a. $a = -10$, $b = -5$, $c = 3$

b. $a = 4$, $b = 6$, $c = -1$

c. $a = 5$, $b = 0$, $c = 7$

d. $a = -2$, $b = -3$, $c = 0$

8. Evaluate the expression $\pm\sqrt{b^2 - 4ac}$ for the given values of a , b , and c :

a. $a = 2$, $b = -7$, $c = -15$

b. $a = 9$, $b = 30$, $c = 25$

c. $a = 1$, $b = 0$, $c = -81$

d. $a = 2$, $b = -3$, $c = 0$

❑ THE ULTIMATE EXAMPLE

For our final example of this chapter, let's evaluate the expression

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{for the values } a = 1, b = -5, c = 6$$

First change every variable to a pair of parentheses — this is not required, but it's a handy little trick to help us avoid errors:

$$\begin{aligned}
 &= \frac{-(\) \pm \sqrt{(\)^2 - 4(\)(\)}}{2(\)} \\
 &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(6)}}{2(1)} && \text{(substitute the given values)} \\
 &= \frac{5 \pm \sqrt{25 - 24}}{2} && \text{(do exponents and multiplying)} \\
 &= \frac{5 \pm \sqrt{1}}{2} && \text{(finish the inside of the radical)} \\
 &= \frac{5 \pm 1}{2} && \text{(the positive square root of 1 is 1)} \\
 &= \frac{5+1}{2} \text{ or } \frac{5-1}{2} && \text{(split the plus/minus sign)} \\
 &= \frac{6}{2} \text{ or } \frac{4}{2} && \text{(simplify the numerators)} \\
 &= \boxed{3 \text{ or } 2} && \text{(and finish up)}
 \end{aligned}$$

There's no homework for this section, but don't be ☹ — there will be plenty of these problems when you study the Quadratic Formula.

Solutions

1. For each quadratic equation, substitute each “solution” (separately) into the original equation. Then work the arithmetic on each side of the equation separately. [Do NOT swap things back and forth across the equals sign.] In all cases, the two sides should balance at the end of the calculations.
2. a. 3, 9, 17 b. 2, -8, 14 c. 6, 2, -2 d. 5, -13, -1
 e. 12, 0, 13 f. 1, -13, 0 g. 1, -1, 1 h. -2, 0, -19
 i. -1, 14, 0 j. -1, 0, -99 k. 2, -4, -3 l. -3, -2, 5
 m. 1, -7, 4 n. 6, -1, -1 o. 18, 0, 0 p. -3, 3, 0
3. a. 145 b. 8 c. 1 d. 25 e. 0 f. -4
4. a. 7 b. -3 c. 7 d. Does not exist e. 8
 f. 1 g. 0 h. 6 i. Does not exist j. 9
5. a. -13 b. 5 c. -7 d. -7 e. 1 f. 0
6. a. 121, -121 b. 11, -11 c. 0 d. Does not exist
7. a. 107 b. -16 c. $\frac{7}{2}$ d. -1 e. 0
 f. 0 g. $\frac{4}{3}, 0$ h. $\frac{8}{9}, -\frac{10}{9}$



8. a. $b^2 - 4ac$

$$= ()^2 - 4()()$$

Converting each variable to a set of parentheses is a handy way to make sure everything is written properly.

$$= (-5)^2 - 4(-10)(3)$$

Note: The parentheses around the -5 are required, because as we've learned, $(-5)^2 = 25$, while $-5^2 = -25$.

$$= 25 - (-120) = 25 + 120 = 145$$

b. 52 c. -140 d. 9

9. a. $\pm \sqrt{b^2 - 4ac}$

$$= \pm \sqrt{()^2 - 4()()}$$

$$= \pm \sqrt{(-7)^2 - 4(2)(-15)}$$

$$= \pm \sqrt{49 - (-120)}$$

$$= \pm \sqrt{49 + 120}$$

$$= \pm \sqrt{169}$$

$$= \pm 13$$

b. 0 c. ± 18 d. ± 3

*“Education is what
you get when you
read the fine print.
Experience is what
you get if you
don’t.”*

– **Pete Seeger**